Edhesive AP Statistics **Unit 10 Test – Solutions**

**Multiple Choice:** Choose the best answer choice for the following problems.

*Questions 1-2 apply to the following data:*

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Trait 1 | Trait 2 | Trait 3 | Totals |
| Group 1 | 121 | 231 | 98 | 450 |
| Group 2 | 76 | 102 | 84 | 262 |
| Group 3 | 88 | 156 | 132 | 376 |
| Totals | 285 | 489 | 314 | 1088 |

1. Suppose the data presented above represents a study in which 450 people were randomly sampled from Group 1, 262 from Group 2, and 376 from Group 3, and were characterized as exhibiting either Trait 1,2, or 3. Which expression gives the correct expected value of people in Group 2 exhibiting Trait 1?

The samples were collected for this test in a way appropriate for a chi-square test of homogeneity, since random samples were collected from different groups. The expected value is (row total)(column total)/N or in this case 262\*285/1088.

1. Suppose the data presented above represents a study in which 1088 people were randomly sampled from a population and were categorized as belonging to either group 1, 2, or 3, and exhibiting either trait 1, 2, or 3. Which expression gives the correct expected value of people in Group 3 exhibiting Trait 3?

The samples were collected for this test in a way appropriate for a chi-square test of independence, since a random sample was collected from one population and two categorical variables were assigned to each participant. Though the method of sampling is different from question 1, the expected values are computed in the same way. So the expected value is (row total)(column total)/N or in this case 376\*314/1088.

*Questions 3- refer to the following situation:*

Benford’s Law is an observation about the frequency distribution of leading digits, and states that in many naturally occurring collections of numbers, the leading significant digit is likely to be small. Specifically, there is a 30.1% chance that the first digit will be a 1, a 30.1% chance the first digit will be either a 2 or 3, a 24.3% chance the first digit will be a 4, 5, or 6, and only a 15.5% chance the first digit will be a 7, 8, or 9. Someone reads about this in their local paper. Intrigued, they look through the rest of the articles looking for numbers. They find 84 numbers in the paper and record the following frequency distribution of leading digits.

|  |  |  |  |
| --- | --- | --- | --- |
| Leading digit = 1 | 2 or 3 | 4, 5, or 6 | 7, 8, or 9 |
| 27 | 25 | 22 | 10 |

1. What is the appropriate null hypothesis to run a goodness-of-fit test?
   1. The proportion of numbers with a leading digit of “1” is the same
   2. These data are independent of the Benford’s Law frequency distribution
   3. These data are not consistent with the Benford’s Law frequency distribution
   4. These data are consistent with the Benford’s Law frequency distribution
   5. The proportions of numbers in each category is the same as Benford’s Law distribution

For a goodness-of-fit test, we are comparing a set of data to a hypothesized frequency distribution and testing for consistency. In this case the null hypothesis is that the data we obtained from the paper is consistent with what is predicted by Benford’s Law.

1. What is the value of the statistic for the goodness of fit test on these data?

For this goodness of fit test, we compute the expected values based on the predicted frequency distribution of leading digits. For a leading digit of 1, for example, we get 0.301\*84=25.284. We then get to a value by using

1. What is the P-value for this test, and at a confidence level of what does that say about the results?
   1. ; These data are not consistent with Benford’s Law
   2. ; These data are consistent with Benford’s Law
   3. ; These data are not consistent with Benford’s Law
   4. ; These data are consistent with Benford’s Law
   5. ; These data are consistent with Benford’s Law

The data has 3 degrees of freedom and a of 0.944, putting the P-value between 0.90 and 0.75. Therefore we do not meet the level of significance required and cannot reject the null hypothesis. Thus we conclude that these data are consistent with the Benford’s Law frequency distribution.

*Questions 6-8 relate to the following situation:*

A dating website conducts a survey on a random sample of its users. One of the questions asks users to rank the importance of finding someone with similar religious beliefs. The results are shown in the two-way table below.

|  |  |  |
| --- | --- | --- |
| Importance of religious compatibility | Male | Female |
| Not important at all | 25 | 17 |
| Somewhat unimportant | 31 | 27 |
| Somewhat important | 32 | 34 |
| Very important | 15 | 12 |

1. What would be the appropriate number of degrees of freedom if a test were to be performed on the dating survey data?

* 1. 2
  2. 3
  3. 4
  4. 7
  5. 8

Degrees of freedom is calculated by (#rows-1)(#colums-1)=1\*3=3.

1. Based on the dating survey data, what is the expected number of men to report that religious compatibility is “Very Important”?
2. 8.01
3. 12.59
4. 13.50
5. 14.41
6. 25.75

Expected values are computed by or in this case 27\*103/193=14.41.

1. In an effort to determine if the importance of religious compatibility is independent of age, the respondents were also broken up into age categories. These categories were “younger than 25”, “25 to 35”, and “older than 35”. How many degrees of freedom would a test of independence have in this case?
   1. 1
   2. 3
   3. 6
   4. 11
   5. 12

We use the same formula from problem 4, but now we have 3 age categories and 4 response possibilities, so df=(4-1)\*(3-1)=3\*2=6.

1. A researcher is conducting an experiment on mice. She runs them through a symmetrical maze with two exits (both paths are the same, eliminating a bias for the easier route) and is interested in learning if there is a gender bias towards different types of cheese. She randomly selects 13 female mice and 14 male mice to run the maze a number of times each and records which type of cheese each prefers based on which exit they tend to favor. Assuming the conditions for a chi-square test for homogeneity are met, which of the following statements about the test is true?
   1. The null hypothesis for the test is that the proportion of each gender who prefer each cheese is ½
   2. Since the sample sizes are not equal, the test is not valid
   3. The test is invalid since she should have randomly selected mice from her lab and recorded gender as a separate categorical variable
   4. The larger the difference in cheese preference between male and female mice, the larger the chi-square statistic is likely to be.
   5. A small chi-square statistic means that each mouse has an equal likelihood of preferring each type of cheese

A large chi-square means it is very unlikely that there is an equal distribution of cheese preferences between mice genders . For the rest, the null hypothesis is that there is no difference in cheese preference between male/female mice, the sample sizes do not need to be equal, (C) describes a chi-square test of independence, and a small chi-square means it is likely there is no difference in cheese preference between male/female mice, but does not say anything about the preference distribution. After all, cheddar is obviously the best cheese.

1. In another lab, a scientist recreates the maze described in the last question, this time to ascertain if age influences cheese preference. For his experiment, he randomly selects 25 mice, runs then through the maze several times each to determine their cheese preference, and then records the preferred cheese as well as the age group of the mouse. Assuming the conditions for inference are met, which of the following tests would be appropriate?
   1. A two-sample t-test for a difference of means
   2. A t-test for a correlation of proportions
   3. A chi-square test of independence
   4. A chi-square test of homogeneity
   5. A chi-square test of goodness-of-fit

A chi-square test of independence is appropriate since 25 mice were randomly selected and two categorical variables were determined for each mouse- in this case age group and cheese preference.

**Free Response – Solutions**

1. An ornithologist conducts a survey of song sparrows and records the number of sparrows as well as what type of tree they were in at the time. The results are shown in the table below.

|  |  |  |
| --- | --- | --- |
| Tree Type | Proportion of tree type in forest | Number of song sparrows observed |
| Oak | 0.24 | 37 |
| Elm | 0.1 | 22 |
| Maple | 0.27 | 41 |
| Pine | 0.39 | 37 |
| Total | 1 | 137 |

* 1. The researcher expects there to be no tree preference for the birds and so expects the number of sparrows observed to be proportional to the amount of that tree observed in that forest. Are these data consistent with that expectation? Assume the conditions for inference are met and conduct an appropriate statistical test.

We compute the goodness-of-fit by first computing the expected values then summing .

|  |  |  |
| --- | --- | --- |
| Observed | Expected | (O-E)^2/E |
| 37 | 32.88 | 0.516253041 |
| 22 | 13.7 | 5.028467153 |
| 41 | 36.99 | 0.434714788 |
| 37 | 53.43 | 5.052309564 |
|  |  | 11.03174455= |

With df=3 and we can see from a chi-square table that 0.01<P-value<0.05. Thus we can claim with 95% confidence that these data do not support the hypothesis that all trees are preferred equally.

* 1. Which type of tree do the song sparrows seem to prefer? Explain.

Based on the table we calculated in part (a), the Elm (with a value of 5.02) has the largest increase in birds observed weighted to those expected.

1. A school is holding elections for class president. A reporter for the school paper conducts a survey ahead of the election and asks a random sample of her classmates whom they were voting for. She recorded who they were going to vote for as well as which grade they were in. Her results are shown in the table below.

|  |  |  |  |
| --- | --- | --- | --- |
|  | 11th grade | 12th grade | Total |
| Bill | 52 | 56 | 108 |
| Steph | 77 | 51 | 128 |
| Total | 129 | 107 | 236 |

If the reporter wants to reach a confidence level of 95%, can she claim that there is a correlation between grade and who they will vote for? Construct an appropriate statistical test, state your hypotheses and display your methods clearly.

We can use the chi-square test of independence to determine if the two variables are correlated. The hypotheses to be tested are:

The variables are uncorrelated

The variables are correlated

We begin by computing the degrees of freedom and expected counts from and .

The expected values are:

|  |  |  |
| --- | --- | --- |
|  | 11th | 12th |
| Bill | 59.03 | 48.97 |
| Steph | 69.97 | 58.03 |

Thus we compute a of

From the chi-square table, using df=1, we see this gives 0.05<P-value<0.10. We therefore fail to achieve a 95% confidence level and the reporter should not claim that there is a correlation between grade and voting preference.

1. The National Aeronautics and Space Administration (NASA) relies heavily on public support for space exploration to ensure funding from the federal government. As a direct nod to this, NASA conducts a survey of a random sample of US residents and asks them which mission they wish to see NASA pursue most; a manned mission to Mars, a robotic mission to drill through the ice on Jupiter’s moon Europa in search of life, or a manned lunar base. For each test participant, NASA records their preferred mission as well as their age group. The data are shown in the table below.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  | Age | | | |
|  |  |  | 21 to 45 |  |
| Preferred NASA Mission | Manned Mars mission | 312 | 303 | 243 |
| Robots to Europa | 198 | 295 | 202 |
| Lunar Base | 96 | 152 | 102 |

A chi-square test was used to determine if there is an association between the preferred mission and the age of the participant. Computer output of the test results is shown below.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | <25 | 25-45 | >45 | Total |
| Mars | 312 (O) | 303 | 243 | 858 |
| 273.23 (E) | 338.15 | 246.62 |
| 5.50 (O-E)^2/E | 3.65 | 0.05 |
| Europa | 198 | 295 | 202 | 695 |
| 221.32 | 273.91 | 199.77 |
| 2.46 | 1.62 | 0.02 |
| Moon | 96 | 152 | 102 | 350 |
| 111.46 | 137.94 | 100.60 |
| 2.14 | 1.43 | 0.02 |
| Totals | 606 | 750 | 547 | 1903 |
| Chi-square=16.91, DF=4, P-value=0.002 | | | | |

1. State the appropriate null and alternative hypothesis for this test.

Mission preference is not correlated to age

Mission preference is correlated to age

1. Are the conditions for a chi-square inference met for these data? Discuss.

A chi-square test of independence requires that:

Sampling be random: condition met, NASA conducted a random survey of US residents

Variables be categorical: condition met, both the mission and the age groups are categorical variables

Expected counts >5: condition met, all expected counts are well over 5.

1. Given the test results, what should NASA conclude?

A P-value of 0.002 means it is extremely unlikely for the null hypothesis to be true. Therefore NASA should conclude that age does play a factor in determining people’s mission preferences

1. Based on your response to part (c), which type of error (I or II) might NASA have made? What does this type of error mean in the context of this test?

A Type I error (rejection of a true null) is possible. Given the low P-value it is very unlikely, but it is possible that NASA is concluding age plays a role in mission preference when in reality there is no difference.

1. NASA wants to engage young people in their mission and decides to pursue the mission most strongly supported by the under 25 age group. Which mission should NASA carry out? Justify your answer.

To answer this we look at the values of the (O-E)^2/E computation to see which mission shows a disproportionately large number of observed tallies. For the <25 age group, this is the Mars mission with a score of 5.5. In fact this is the only mission where the observed counts were higher than the expected counts. Therefore NASA should pursue a manned mission to Mars. Really they should.